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Sensor-realistic simulation of images in diffraction-limited imaging systems

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Abstract: Optical imaging is the building block of different measurement systems, whose imaging quality is influenced by many factors including the choice of imaging parameters, optics, and the imaging sensor. Realistic simulations can therefore be very beneficial for automatic optimization of such degrees of freedom. In this paper, we present a physically-based model for the image formation consisting of three parts: geometrical light transport simulation using available computer graphics methods, introducing lens imaging effects using Fourier optics, and integration of the EMVA 1288 sensor data. The proposed model has been utilized for simulating the images with speckle interference patterns caused by emitting a laser line on a metal surface. The agreement of the simulations with real images demonstrates that the model can produce accurate approximations to the real images, especially for diffraction-limited systems. We also discuss the simulation process for both coherent and incoherent imaging cases to simulate imaging systems having one or both types of illuminations.

Keywords: Physically-based simulation, Fourier optics, EMVA 1288, diffraction-limited, speckle, laser.

1 Introduction

A wide variety of inspection and measurement tasks are nowadays performed by means of imaging systems [2]. The reliability of such systems depends on numerous factors, including light sources, scene geometry, scattering properties of the surfaces, quality of the optics, and the electronic sensors. A combination of these factors eventually influences the resulting image and leads to some uncertainty in the final measurement [14]. The complex nature of the image formation process makes it difficult to evaluate the performance of an optical measurement without taking real images, and it is impractical to experimentally try out all imaging conditions for each setup design. Therefore, reliable simulations can provide a solution for automatic evaluation and eventually optimization of imaging setups.

Physically-based rendering [15], as a subfield of computer graphics (CG), involves physical simulation of the light transport in the scene within the limits of geometrical optics. Utilization of physically-based rendering for simulating optical systems has been a recent research direction [16] with the focus of correctly predicting the resulting radiance on a virtual sensor, which can be further used to calculate the accumulated energy on the sensor. Thanks to the widely adopted EMVA standard 1288 [8], it is now...
possible to further link the accumulated energy to the digital response of a sensor, i.e. the resulting digital image, to evaluate the performance of a particular imaging sensor.

For sufficiently realistic simulation of optical systems that operate close to their fundamental resolution limit [11], diffraction effects of the imaging aperture play an important role. Especially for the case of coherent imaging, diffraction leads to the formation of an interference pattern called speckle [3], which significantly influences the measurement uncertainty [4]. The resolution of an incoherent imaging system is also influenced by diffraction. Such effects are either not describable by geometrical optics, or others such as aberrations and defocus, impose further computational complexities to the ray tracing.

In this paper, we introduce a physically-based image formation model for simulating an imaging system, consisting of three building blocks: light transport in the scene, optical imaging in the lens, and conversion to a digital image on the sensor. The main contribution of this paper is the inclusion of the last two steps to the conventional simulation process as post-processing. To verify the model, we considered simulation of the images in a simple laser triangulation scenario with a flat piece of surface. The agreement of simulated and real images demonstrates that the combination of geometrical optics simulations of light transport in the scene, Fourier optics modeling of the imaging system, and sensor EMVA specifications, make it feasible to reproduce a good approximation of the actual signal to noise ratio in real images, especially for diffraction-limited optical systems. We also discuss general ideas regarding the simulation process for both coherent and incoherent imaging systems, which can be utilized for realistic simulation of imaging systems having one or both types of light sources.

2 Image formation model

In this section, the process of image formation in an imaging system is modeled in three main steps. Section 2.1 gives a brief overview of physically-based ray tracing methods from the field of computer graphics. Sections 2.2 and 2.3 introduce the further steps for sensor-realistic simulation of the images in an optical imaging system. Figure 1 depicts a simplified schematic of the image formation process.

2.1 Light transport

Light transport in the scene is influenced by a complex combination of many phenomena. Based on the geometry and optical properties of the participating objects, the emitted light can get absorbed, transmitted, or scattered multiple times before reaching the camera. The focus of physically-based rendering in CG is to trace the rays (usually the reverse way starting from the camera to the light source for rendering efficiency) in a physically correct way, all within the limits of geometrical optics. This requires a sufficient knowledge of the scene geometry as well as the optical properties of the surfaces, such as the Bidirectional Reflectance Distribution Function (BRDF) [5].

Rendering algorithms rely on Monte Carlo or Markov Chain Monte Carlo methods for sampling the rays and estimating the light transport, with their difference being only the approach towards the stochastic path sampling [16] to increase the rendering efficiency in different scenarios. Retzlaff et al. [16] discuss different rendering algorithms and their suitability for simulating optical measurement systems. An accurate physically-based rendering can lead to a good approximation of the radiance on the sensor, in particular with the assumption of a pinhole camera model.

Estimation of light transport is an interesting field with a rich literature, and there are some readily available physically-based renderers, such as Mitsuba [12], providing...
This is technically possible, however, with an increase of the rendering time. In addition, interference patterns caused by coherent light cannot be reproduced in this way and require other phenomenological approaches [1], or dedicated optical simulation methods [7]. In this work, we introduce a systematic way towards simulation of such optical effects with a special focus on diffraction.

2.2 Imaging optics

The role of an optimal imaging is to focus the received light through the aperture coming from one point, into another point on the sensor. In reality however, this mapping is not optimal and we expect deviations from an ideal image caused by aberrations, out-of-focus effects, aperture diffraction etc. Some of these deviations have been already considered in geometrical ray-tracing frameworks by including lenses and tracing rays through them [10]. Other phenomena like aperture diffraction, only explainable by wave behavior of light, have been also simulated in a geometrical ray-based manner [13, 18]. Ray-tracing in real lenses for sufficiently mimicking such effects can significantly reduce the rendering efficiency and increase the rendering time. In addition, interference patterns caused by coherent light cannot be reproduced in this way and require other phenomenological approaches [1], or dedicated optical simulation methods [7]. In this work, we introduce a systematic way towards simulation of such optical effects with a special focus on diffraction.

In this part, we first briefly discuss the integration of light over the aperture, and later propose using frequency filtering methods from Fourier optics to introduce imaging phenomena caused by the lens and aperture.

2.2.1 Aperture integration

The common goal of typical path or ray tracing algorithms is to correctly estimate the scene radiance \( L_s \) from a point \( p \) to the corresponding sensor point \( x_p \), along the main ray direction, as if a perfect pin-hole camera images the scene. As illustrated in Fig. 2, the actual incoming irradiance from the scene to the imaged point requires an integration of radiance values received by the aperture over the subtended solid angle \( \omega \). In computer graphic terms, the Monte Carlo sampling not only requires to sample positions on the sensor, but also on the aperture. This is technically possible, however, with an increase of the rendering time.

In many imaging scenarios, the distance of the object to the aperture is considerably larger than the aperture diameter, and therefore, the solid angle subtended by the lens is small. In such cases, and especially if the surface is rather diffuse than specular, we can approximate the scene radiance to be almost uniform over the whole solid angle \( \omega \). With this assumption, the irradiance collected by the aperture is approximated as [17]

\[
\text{Irr}(x_p) = \frac{\pi}{4} \left( \frac{D}{f} \right)^4 \cos^4 \theta L_s(x_p), \tag{1}
\]

where \( f \) corresponds to the focal length, and \( D \) denotes the aperture diameter.

2.2.2 Simulating optical effects using Fourier optics

In Fourier optics, an optical system is modeled as a linear system with a Point Spread Function (PSF) [9]. The definition of this linear system, however, depends on the coherence of the light source. If the illumination is incoherent, the system is linear on the input intensities \( I_{in} \). Thus the output intensity \( I_{out} \) for incoherent imaging is given by the convolution with the incoherent PSF \( h_i \)

\[
I_{out} = I_{in} \ast h_i. \tag{2}
\]

If the illumination is spatially coherent (e.g. laser light), the system is no more linear on the input intensities, but rather acts linearly on the input wave field \( E_{in} \) by convolution with the coherent PSF \( h_c \)

\[
E_{out} = E_{in} \ast h_c. \tag{3}
\]

In contrast to intensities which are positive real scalar values, the input wave field is a complex term \( |E|e^{j\angle E} \), whose amplitude is proportional to square root of the input intensities \( |E| \propto \sqrt{I} \). For correctly simulating interference patterns which appear in coherent imaging systems, the phase of the input wave \( \angle E \) is the deciding factor. If a common phase is set for the whole wave field (usually zero phase), coherent formulation reduces to the incoherent case. When coherent light is scattered by a surface which
is rough in wavelength scale, the phase factor can be assumed to be independently uniformly distributed in $[0, 2\pi]$ [1]. The system PSF in coherent imaging can lead to highly nonlinear effects on the output intensities, as given in Eq. 4. The nonlinear diffraction effects for a laser light source appear as speckles.

$$I_{c}^{\text{out}} = (\sqrt{\text{Im}e^{i\varphi}} * h_{c})^{2}$$

Computationally, it is more convenient to translate this systematic perspective in the equivalent one in the frequency domain. The Fourier transform of $h_{c}$ is called the Amplitude Transfer Function (ATF) and in incoherent case, the Fourier transform of $h_{c}$ is denoted by the Optical Transfer Function (OTF). It has been shown that OTF is the normalized autocorrelation function of the ATF [9].

For both formulations, $I_{c}^{\text{in}}$ can be given by an ideal image predicted by graphical rendering methods. In principle, knowing the exact AFT or OTF, additional imaging phenomena can be introduced to the simulated images by an image processing step. We usually can only approximate the system transfer function, and therefore, the optical effects will be also approximative. In the ray tracing step, it is important to keep track of coherent and incoherent light components on the sensor, and induce a random phase to the coherent rays which reach the sensor after being scattered by a rough surface. Coherent and incoherent image components must be differently filtered and then added together.

### 2.2.3 Diffraction-limited imaging systems

Diffraction of the imaging aperture, imposes a fundamental resolution limit on optical systems, the Rayleigh limit [9]. If other imaging artifacts, such as aberrations, are not dominant, and the imaging sensor has already a better resolution than the Rayleigh limit, the system is known to be diffraction-limited and operates at (or close to) this resolution limit. In this case, the ATF or OTF of an aperture diffraction is enough to model the whole imaging objective, which can be calculated given the aperture shape, size, and its distance to the sensor. For instance, the ATF of a circular aperture with diameter $D$ and distance $z$ to the sensor, is given by

$$H(f_x, f_y) = \text{circ} \left( \frac{2\lambda z}{\sqrt{f_x^2 + f_y^2}} \right),$$

where $(f_x, f_y)$ denote the 2D frequency components and

$$\text{circ}(x) = \begin{cases} 1, & \text{for } |x| \leq 1 \\ 0, & \text{otherwise} \end{cases}.$$
Fig. 3: EMVA 1288 model for characterization of image sensors [8].

whose average value and noise characteristics are also described in the standard. The input to this model is the noisy (known as shot noise) number of photons reaching a pixel area, where only a percentage of them can be converted to electrons (quantum efficiency). The electrical voltage induced by the electrons is subject to a Gaussian dark noise. This voltage is amplified by a system gain $K$ and then quantized to a digital gray value. Figure 3 shows an overview of the EMVA 1288 model. For more information in this regard can be found in the original standard [8].

In this work, the EMVA specifications of the used sensor are taken into account, which led to a good approximation of the digital intensities on the rendered images.

2.4 Discussion on the model

To reach a sufficient realism in the simulations, at least the dominant imaging effects must be correctly simulated. The proposed image formation model, can best work for diffraction-limited optical imaging systems, especially those operating with laser light. The speckle patterns caused by laser sources usually play the role of the limiting factor for the measurement precision [4], especially for large f-numbers (i.e. small aperture area). Increasing the aperture area typically increases the resolution of the imaging system and moves the system away from being diffraction-limited. In these cases, other components of the imaging system play a more dominant role and therefore, require a precise simulation. For example, if speckles in coherent imaging are too fine (small f-numbers), surface roughness texture can also influence the accuracy of the laser line detection, whereas with large speckles, intensity variations caused by the surface height profile are often not resolved and can be neglected.

Although so far not directly evaluated, the Fourier optics methods proposed here, can be used to integrate further imaging effects, such as aberrations and defocussing into simulations, however, the filtering process can be more challenging and requires further analysis.

3 Case study: Imaging a laser line

In this section, we consider imaging a laser line illuminating a flat piece of a metal surface, and physically simulate the image formation process as a proof of concept for the model proposed earlier. Figure 5 displays the experimental setup for taking images, in which a camera is looking perpendicularly on a surface being illuminated by a laser source. The laser with a measured power of 23 mW and a Powell lens distributes the laser power almost uniformly on a projected laser line. The camera and laser distance to the surface, as well as the laser incident angle have been calibrated before taking images.

We have followed the description of Eichler and Eichler [6] for modeling the propagation of laser light and calculating the irradiance profile on the surface. For the laser with a Powell lens, the intensity distribution is uniform along the length, and Gaussian in the cross section of the beam, whose waist can be calculated given the distance and laser specifications (wavelength, divergence, focal length) [6]. Using this formulation, we simulated the irradiance profile of the laser on the surface with a resolution of 10 µm.

As shown in Fig. 4, the light transport in this scenario is rather simple, because the flat surface causes no multiple reflections in the scene and it suffices to only simulate the light transport from the laser to surface, and further a one-step scattering from the surface into the camera (direct light illumination), without Monte Carlo based path or ray tracing. This scenario has been intentionally
chosen simple, so as to more focus on the Fourier optics modeling and integration of the EMVA sensor data. The BRDF of the surface has been previously measured and further used to estimate the scattered radiance to the camera. We further used Eq. 1 for estimating the total irradiance reaching the sensor, holding the assumption of a uniform radiation over the aperture, as the surface distance is much larger than the aperture diameter and the camera looks on the surface mostly through diffuse light. Diffuse part of a BRDF usually does not contain abrupt variations.

Using the methodology described earlier, the imaging system was modeled as a diffraction-limited coherent optical system with a circular aperture, and the diffraction effects (leading to the formation of speckles) were simulated according to Eq. 4, by setting $\varphi$ to an independently generated random phase. We further estimated the number of photons on each pixel sensor, using Eq. 7, and applied the camera EMVA specifications to transform irradiance into digital intensities.

For imaging, a color camera with available EMVA 1288 datasheet (BFLY-PGE-23S6C-C series from FLIR) and a 16 mm Fujinon objective was chosen. Although for laser triangulation applications typically a monochromatic camera is more suitable, we chose to verify our model with a color camera for two reasons: Firstly, in a color camera we get three channels with typically different intensity levels, which allows us to compare the simulation results for different levels of the signal. Furthermore, we can verify the model for simulations in a more general imaging scenario with an additional simulation of the camera’s color filter array (in this case the Bayer pattern). To directly compare the images with simulation results, an image acquisition driver for the camera was developed, in order to fully control all imaging parameters and post-processings such as image gain and white balance correction, and get the raw images that could directly correspond to the simulations after applying the EMVA 1288 model.

### 3.1 Results

Figure 6 compares simulation results with the corresponding real images taken in the experimental setup, for different combinations of aperture and exposure time. The results have been directly obtained by applying the simulation process described earlier, without any further intensity scaling. Increasing the aperture area (from image a to e), leads to a gradual reduction of the speckle size, as the diffraction PSF width also gradually shrinks according to Eq. 5. This behavior has been also reproduced in the simulations. Figure 7 illustrates an average laser intensity profile (averaged over all the image columns) against the same profile on the real image, for the red, green, and blue channel of the camera.

The rendering time was about 2 seconds for each image on a PC with an Intel(R) Xeon(R) CPU @3.7GHZ and 16GB of RAM, however, this time can of course increase when applying CG Monte Carlo methods for simulating the light transport in complicated scenes.

### 3.2 Conclusion and discussion

The results of this paper confirm that modeling the aperture diffraction using Fourier optics methods can produce a good approximation of the actual speckle granularity and contrast. Simulations of Depth-of-field and aberran-
Fig. 6: Simulated vs. real images of the experimental setup, with a sensor distance of 0.43 m to the surface, $f = 16$ mm focal-length objective, 45° triangulation angle, and 23 mW laser power. Images with varying aperture diameter $D$ and exposure time $t$:

- a) $D = f/22$, $t = 811$ ms
- b) $D = f/16$, $t = 202$ ms
- c) $D = f/11$, $t = 12$ ms
- d) $D = f/8$, $t = 25$ ms
- e) $D = f/5.6$, $t = 6.3$ ms

Fig. 7: Averaged profile of the simulated laser line against the real image for the red, green, and blue channel of the camera. The profiles correspond to image b in Fig. 6.

Applying the EMVA 1288 enabled us to closely predict the spectral response of the camera to the incoming light. Therefore, in addition to the imaging parameters such as the exposure time and the aperture, one can also evaluate different potential camera sensors and choose the one which best suits the imaging application. It is also important to note that the reported EMVA 1288 data for a camera can manifest slight deviations from unit to unit. Therefore, for an exact simulation of a sensor it is recommended to measure the EMVA parameters of the intended sensor.

The best agreement between simulated and real images have been achieved for f-numbers 16 to 8. For smaller f-numbers, the imaging system is gradually less diffraction-limited and the simulations show a slightly better SNR than real images. In the case of f-number 22, the speckle sizes on the real images appear to be slightly bigger than the ones in the simulations. We assume this can be associated to the approximation of the aperture shape, which is in fact not a perfect circle. We expect improvements by taking the actual shape of the aperture into account.

Although simplified simulations of a complex physical phenomena, such as optical imaging, can always contain discrepancies compared to real images, these simulations can still be beneficial for the design process by providing the expert with good initial configuration suggestions and avoiding unnecessary experimental work and hardware costs.

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References


