Towards Surface Inference in Industrial Inspection

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**Abstract:** Automated product inspection play an important role in today’s manufacturing process, and therefore, the design of optimized and precise measurement setups are a requirement for efficient product quality assurance. Due to the high dimensionality of the design space, a manual choice of the geometrical and optical parameters is associated with high costs, tedious experimental work, and often non-optimal results. Thus, automatic planning methods which seek to optimize the setup degrees of freedom for a particular measurement are of special importance in this field. For automatic evaluation of an inspection, there exist typical evaluation metrics including but not limited to, the measurement uncertainty and the scan resolution. However, it is often not trivial how to combine different optimization criteria to optimize the setup based on the requirements. For example, it is not obvious how to compare an the result of an inspection with a high lateral resolution and a high uncertainty against another inspection, with a low lateral resolution but precise measurements. We propose to fuse the metrics through a probabilistic surface inference to quantify the amount of information gained by a specific setup configuration. To this end, we model the surface by a Gaussian random process and introduce a local surface inference method based on the local surface orientation. The measurement points delivered by a laser triangulation setup are simulated using real-time graphical simulations, and the uncertainty of single point measurements are estimated. This data is further used as input to the local inference method. The inference results are demonstrated for the inspection of the intake manifold of a cylinder head.

1 Introduction

In today’s manufacturing processes, machine vision provides industries with innovative solutions towards the automation of the production quality assurance
Typical inspection techniques include laser triangulation, fringe projection, and deflectometry, as well as confocal microscopy and interferometry for high precision measurements \cite{BLF15}. Different solutions are primarily chosen based on the inspection demands and the target surface properties. Based on the chosen technique, there are further degrees of freedom in the inspection setup, including among others, the position and orientation of the camera(s) and the illumination source(s), as well as the optical configuration of the participating devices \cite{MBI16b}, which need to be set by the expert.

As the design space is high dimensional, a manual setup design necessitates a tedious trial and error process and a lot of engineering experience, which is often associated with high costs and can easily end up in a compromise between contrary requirements, or resort to a rule of thumb setup design. Moreover, to quantitatively evaluate the inspection, one needs to experimentally evaluate the inspection performance based on the specific target product and the available sensors, similar to the work by Feng et al. \cite{FLX01}. One of our intentions in this report is to derive a suitable comprehensive cost function based on the inspection requirements, to automatically design optimized machine vision prototypes for inspecting different products without the need to realize the setup. This prevents unnecessary costs and the tedious experimental work.

To automate the setup design process and to plan an optimized inspection, several sensor planning methods have been introduced in the literature \cite{TTA95, Sco09, GO16}. In these works, different evaluation metrics are considered and taken as optimization cost functions into account. The optimum surface coverage \cite{TG95} is one of the main goals pursued in almost all planning methods. Other constraints such as depth of field, field of view, and resolution are also of interest \cite{TTA95}. More recent works also take the uncertainty of measurement into account \cite{Sco09, GO16, MBI16a}.

For incorporating the metrics into an optimization framework, a few approaches have been proposed. One approach is to evaluate the metrics as constraints and try to fulfill them simultaneously \cite{TTA95}. Another approach is based on the computation of the measurability matrix, introduced by Tarbox \cite{TG95}. This method is primarily based on a matrix data structure whose rows span the discretized surface points and columns span the discretized sensor view-points, and each matrix element indicates boolean fulfillment of the inspection constraints \cite{Sco09} or a weighted sum of different inspection metrics \cite{GO16}. A third approach is to consider different metrics as separate criteria optimized in a multi-objective optimization framework \cite{DO05}.

The evaluation criteria can be contradictory. This turns a multi-objective optim-
mization more into choosing a trade-off between the goals. For example, increasing the distance leads to a broader sensor coverage but also increases the measurement uncertainty. Finding a suitable weighting between the goals is also not trivial and can easily lead to one metric dominating the other. For instance, it is difficult to compare an inspection with a high resolution but a high measurement uncertainty with another inspection with a lower resolution but also a lower uncertainty of measurement. For a dense sampling of the design space which is often high dimensional, the memory allocation demand for the measurability matrix is also prohibitive.

Our solution for a suitable metric fusion is to formulate the amount of information we obtain by each measurement, using a Bayesian inference framework. In this method, the measurement resolution and the corresponding uncertainties of the point measurements are fused together by the surface inference method. To this end, we model the target surface as a random process and adapt the problem to a Gaussian Process (GP) inference \[RW06\]. The measurement points delivered by each sensor view-point are simulated using real-time graphical simulations, augmented with an estimate of their corresponding measurement uncertainty, and further used in a GP learning framework to update the random process. The standard deviation of the updated random process (a posteriori process) actually indicates the amount of information that a specific sensor configuration delivers to us, and can be evaluated at each arbitrary point on the surface. The probabilistic surface inference method can be applied to all point-based inspection techniques such as the general group of structured-light scanning methods \[BLF15\].

2 Surface Inference

Most visual inspection techniques scan the target object through sampling the surface in a number of 3D points. Among different criteria discussed in the previous section, the quality of the scanned point cloud can be mainly characterized by the *scan resolution* and the respective *uncertainty* of each measured point \[MBI+16b\]. Figure 2.1 shows a 2D illustration of inspecting a surface with a number of point measurements, where each ellipse represents the uncertainty of each sampled point. Before we begin with formulating the surface inference, we briefly introduce these two properties.

**Scan resolution** indicates the local density of the points on the surface, or the lateral resolution of the measurement. Based on the object geometry and the relative distance of the surface points to the camera, the surface is in general inhomogeneously sampled. There can be parts unreachable by the illumination or
occluded to the camera, which cause a local point density of zero. Therefore, the
scan resolution property is an extended visibility metric, which not only encodes
the measurability of a surface part, but also its lateral resolution.

**Measurement uncertainty** is defined as the standard deviation of the measurement from the nominal value, which is an indication of the measurement reliability [gum08]. The ellipses in Figure 2.1 correspond to the uncertainty of each measurement point. In the case of 3D measurements, each point is subject to a 3D uncertainty. In our previous work [MBI+16a], we have formulated a framework for probabilistic modeling and analytic propagation of the sources of uncertainty pertinent to a laser scanner. The sources of uncertainty include the uncertainty of the feature detection on the image, the setup geometrical positioning uncertainties, and the optical calibration uncertainties. We have approximated the resulting 3D uncertainty in each point measurement as a Gaussian distribution which corresponds to an ellipsoid (i.e. $3 \times 3$ covariance matrix).

The objective of surface inference is to merge these metrics and determine a probability distribution for the position of an arbitrary point $x_*$ on the surface (see Figure 2.1), given the measurement data. In order to consider the contribution of the measured points to our degree of belief at the point $x_*$, we apply a Bayesian framework. In such a framework, we always have a prior knowledge about the subject under study in the form of an a priori distribution [BS94]. In this case, the a priori distribution can be given based on the typical variations in the manufactured products. As new observations are made, our prior belief will be updated, and an a posteriori distribution is calculated based on the Bayes' law.

The a posteriori standard deviation $u(x_*)$ indicates our remaining uncertainty at the point $x_*$ after the surface has been measured. This way we can evaluate an

![Figure 2.1](image-url): 2D illustration of surface inspection with noisy measurements. Red ellipses denote measurement uncertainties. $x_*$ is an arbitrary point on the surface to be inferred, and $u(x_*)$ indicates the inference uncertainty.
inspection in terms of the amount of the new information it delivers to us about the surface compared to our prior knowledge. In this paper, we refer to \( u(x_*) \) as the \textit{inference uncertainty}, which is in concrete words defined as the standard deviation of our prediction of the surface height \( h(x_*) \) at point \( x_* \).

Different constellations of the inspection setup deliver different sets of sampled points with different measurement uncertainties \cite{MBI16}, which in turn result in different a posteriori distributions for the product surface. Based on the desired inspection requirements, one can evaluate the resulting a posteriori distributions along the surface and use it as an optimization criteria for the setup design.

Modeling each surface point as a random variable indicates that the whole surface is modeled by a stochastic process. A stochastic process is a collection of random variables \( \{h(x)|x \in D\} \) indexed by a set \( D \) \cite{GWB97}. For a surface in 3D, \( D \) is a subset of the \( \mathbb{R}^2 \) space and \( h(x) \) is a random variable corresponding to the height of the continuous 2D input \( x \).

In a 3D inspection, we measure the surface in a number of points \( \{\tilde{x}_i, \tilde{h}_i\} \) with \( \tilde{x}_i \in \mathbb{R}^2 \) and \( \tilde{h}_i = \tilde{h}(\tilde{x}_i) \in \mathbb{R} \). Each point measurement is associated with a measurement uncertainty (covariance matrix) \( \Sigma_i \), depicted as a red ellipse in Figure 2.1. We further define the pair \( \{\tilde{X}, \tilde{h}\} \) as a compact notation for referring to all measurement points. In machine learning terms, the measured data \( \{\tilde{X}, \tilde{h}\} \) is referred to as the training data. Inferring an arbitrary point \( x_* \) denotes computing the probability distribution \( p(h_*|\tilde{X}, \tilde{h}) \) \cite{RW06}.

We distinguish between two different uncertainties in Figure 2.1. The red ellipses correspond to the measurement uncertainty due to the limited precision of the sensor. However, \( u(x_*) \) corresponds to the standard deviation of our prediction (our degree of belief) for the height of the point \( x_* \). The calculation of the a posteriori distribution of the surface using Gaussian Process is the main topic of the next section.

### 2.1 Surface Inference by Gaussian Process

A Gaussian Process (GP) \cite{RW06} is a special variant of a stochastic process, and is widely used in many problems dealing with non-parametric function learning. In a Gaussian process, the joint distribution between each subset of \( \{h(x_1), ..., h(x_k)\} \) is a multivariate Gaussian distribution \cite{RW06}. This property gives the GP the nice feature to have a closed form solution for the desired conditional probability \( p(h_*|\tilde{X}, \tilde{h}) \), using the standard Gaussian conditioning. This is one of the main reasons for the GP popularity in regression and classification problems.
A GP can be fully specified by its mean $\mu(x)$ and covariance function $c(x, x')$:

$$
\mu(x) = E[h(x)], \\
c(x, x') = E[(h(x) - \mu(x))(h(x') - \mu(x'))],
$$

where $E$ denotes the expectation operator. For a GP inference, the mean and the covariance function should be already given or assumed.

The covariance function and its free parameters are central in determining the GP. The type of the covariance function is usually chosen based on our understanding of the properties of the underlying function, such as its differentiability. The free parameters of the covariance function, (often referred to as the hyper-parameters [RW06]), control further properties such as the level of smoothness and the scale of the function. It is common to consider many processes as isotropic stationary processes and define the covariance function as a function of the Euclidean distance $c(r) = c(|x - x'|)$, or non-Euclidean distances. A comprehensive study of covariance functions is given by Rasmussen and Williams [RW06].

The desired a posteriori distribution $p(h_*|\tilde{X}, \tilde{h})$ is another Gaussian distribution, derived by conditioning on the training data. The mean and the variance of the a posteriori distribution at the point $x_*$ can be derived as given in Eq. (2.1) and (2.2) [RW06]. These equations are derived using the standard Gaussian conditioning.

$$
\mu_* = \mu(x_*) + k_* [K + \sigma^2_h I]^{-1}(\tilde{h} - \mu(\tilde{X})) \quad (2.1) \\
\sigma_*^2 = k_{**} - k_* [K + \sigma^2_h I]^{-1}k_*^T \quad (2.2)
$$

In the above equations, $K \in \mathbb{R}^{N \times N}$ is a square matrix containing the pairwise covariance between $N$ training data, $k_* \in \mathbb{R}^{N \times 1}$ is a vector containing the covariance of the inference point $x_*$ with the training data, $I$ is the $N \times N$ identity matrix, and $k_{**}$ is a single scalar given by $c(x_*, x_*)$, which is actually the variance of the prior distribution $p(h_*)$. The training points $\tilde{X}$ in these equations are considered noise-free, and their corresponding function values $\tilde{h}$ are subject to an independent identically distributed (i.i.d.) noise with variance $\sigma^2_h$. The assumption of noise in Gaussian process is somehow different from that of our surface measurement, and for this reason we use the Noisy Input Gaussian Process (NIGP) method [And14] to integrate the 3D uncertainties in the GP.

In the application of surface inspection, the CAD product model can be directly used as the mean function $\mu(x)$. The a priori distribution for an inference point
can be chosen based on the expert knowledge about the production variations (e.g. 10 mm standard deviation compared to the CAD model). Figure 2.2 shows the CAD model of a cylinder head.

The choice of the covariance function is however more complicated. It is important to note that the real surface to inspect is not the ideal CAD model of the product, but rather a complicated surface with different levels of detail. Figure 2.3 shows an illustration of a manufactured surface. This surface is ideally a plane; however, the manufactured surface can contain deviations from an ideal plane in several scales. The inspection resolution is actually the criteria to distinguish between the surface defects, which the inspection is supposed to identify, and the unavoidable surface roughness, which is not supposed to be detected as an error. It is therefore important to choose the covariance function which suits reconstructing the surface upto the desired level of detail that we intend to inspect the surface with. This is usually achieved by suitably setting the parameters of the covariance function. The squared-exponential (SE) or the Gaussian function given in Eq. (2.3) is a very common choice for the covariance function in many applications. Many real rough surfaces are also shown to have a Gaussian autocorrelation function [BS87].

![Surface defects at several levels of detail, in comparison to the ideal flat model.](image)
\[ c(r) = \sigma_f^2 e^{(-\frac{r^2}{2\ell^2})} \] (2.3)

Most covariance functions are parameterized with at least two parameters. One parameter is a scale factor (in this case $\sigma_f^2$) indicating the maximum value of the function, or $c(0)$. This value should be set to the prior variance of the surface, or the variance of the a priori distribution, since we have $c(0) = c(x^*, x^*) = \text{var}(x^*)$. Another parameter is the correlation length-scale (in this case the parameter $\ell$), which indicates the decaying behavior of the covariance function. This length-scale parameter mainly determines the level of detail reconstructed from the surface. Large length-scale values lead to a surface reconstructed with lower frequencies, whereas smaller values tend to reconstruct more details from the surface.

### 2.2 Local Inference

In this section we propose a local inference method for the inference of industrial product surfaces. As seen in equations (2.1) and (2.2), the calculation of the a posteriori mean and variance depends on the calculation of the inverse of an $N \times N$ matrix $K + \sigma_h^2 I$, with $N$ being the number of the training points, i.e. measurement points, which is often a large value. Moreover, the inference variance $\sigma_*^2$ is always calculated along a particular axis, which is actually the axis along which the surface height $h$ is defined. However, industrial products have surface complexities which make it almost impossible to define a function $h(x)$ which can define the whole surface; however, industrial surfaces can be locally well parameterized. As the last motivation for the local inference method, we are particularly interested to derive the surface inference uncertainty locally along the normal direction of the surface to better encode the deviations from the surface, and thus, we are interested in local transformations which rotates the surface along the local normal direction.

We justify the idea of a local inference by referring to the fact that all covariance functions decay monotonically with distance, except for some special cases like periodic functions which are not of interest for us. Therefore, for the inference of a point $x^*$, only the measurement points inside a neighborhood radius $r$ from the point can effectively contribute. The value of $r$ can be directly derived from the covariance function, based on a low threshold, which directly depends on the correlation length-scale discussed previously. This brings us to the idea of a local inference, in which for each inference point of interest, we search for the measurement points inside the neighborhood and derive the inference based on the retrieved points, instead of considering all the points which actually do not
contribute much. The local inference significantly improves the runtime performance, as only a limited set of effective points are considered for each inference. In addition, we are also able to apply the desired local transformations on the retrieved points.

For applying the local transformation, we transform the set of points selected in a neighborhood to a new local coordinate frame by a rotation $R$, so that the transformed heights of the point $h'$ align with the direction of the local surface normal vector. A 2D illustration of this transformation is depicted in Figure 2.4.

The 3D ellipsoids $\Sigma_i$ corresponding to the measurement uncertainty of each point will be also represented in the new coordinate frame by $R\Sigma_iR^T$. Consequently, by applying the GP inference in the new coordinates, the inference uncertainty $u(x_*)$ will be calculated along the $h'$ direction which is parallel to the normal direction. This is desirable since the uncertainty will be always calculated along the local normal direction, indicating the deviations from the nominal model, independent of the complex changes of orientation along the whole surface. The neighborhood sphere in Figure 2.4 moves along the surface and at each point a local transformation and thus a local inference is applied. For the integration of the 3D measurement uncertainties in the GP, we use the Noisy Input Gaussian Process (NIGP) method [And14].

Figure 2.4: 2D illustration of the local inference. The local points are transformed towards the surface normal direction. $h'$ will be parallel to the normal $n$ and the $x'$ axis will be parallel to the local linear approximation of the surface.
3 Inference of a Cylinder Head Surface

In this section we apply the proposed probabilistic surface inference method to evaluate the inspection of a cylinder head using the laser triangulation method. The cylinder head is a geometrically complex product requiring a precise verification, and thus, it is considered as a challenging example for the planning problem. The cylinder head surface is in general not a differentiable surface; moreover, different parts of the model can be manufactured independently which indicates that the defects at some surface parts are not correlated to each other. Therefore, we have manually segmented the model into partially differentiable regions using the Blender software [Ble16], and apply the surface inference separately for each region. Figure 3.1 displays the segmented intake manifold of the cylinder head. Each color represents a separate segment whose inference is done separately.

Before we apply the inference, we simulate the image acquisition from each desired viewpoint and obtain the corresponding measured point cloud of the target. To simulate the image acquisition process, we use the Rasterization Simulation Library (RSL), which was also introduced in our previous report [BP15]. The graphical simulations are GPU-accelerated and can be performed very quickly even on commodity hardware. The simulation input includes the geometrical setup constellation, the laser properties such as wavelength, focal distance, divergence, and opening angle, the camera resolution, and above all the CAD model of the workpiece and frequency of image acquisition during the object movement. By the graphical simulations, we are able to obtain a 3D point cloud corresponding to each complete scan of the object, as well as single frame rendered images. Figure 3.2(a) displays the parameterization of the setup geometrical degrees of freedom in the RSL. Figure 3.2(b) is captured from the RSL environment and corresponds to the simulation of the cylinder head inspection for a particular constellation. More information regarding the physically-based simulation of a laser line and the RSL is given in our previous works [BMI+] [MBI+].
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We further augment the simulated points with their respective 3D measurement uncertainties, based on our previous work \cite{MBI16}. In this work, the sources of uncertainty in a laser triangulation measurement are modeled and propagated to the output. We have considered a 0.2 $px$ standard deviation for the laser detection on the image and a 0.5 $mm$ standard deviation for the positioning of the whole setup (neglecting rotational and optical calibration uncertainties). This will be the input to the local inference method described in section 2.2.

We have applied the proposed inference method on the intake manifold of the cylinder head. Figure 3.3 shows the resulting uncertainty of the inference (standard deviation of the GP), for the inspection of the cylinder head corresponding to the constellation in Figure 3.2(b). We sample the model using the Poisson disk sampling method \cite{CJW09} to achieve sample points which evenly cover the surface, and the local inference method is then applied to each sampled point. For the GP inference, the SE covariance function is used and an a priori standard deviation ($\sigma_f$) of 10 $mm$, and a correlation length-scale of $l = 0.6 mm$ is considered for the surface. The scan simulation contains 700 frames in which the cylinder is moved by 1 $mm$ per frame. The simulated camera captures $512 \times 512$ $px$ images with a vertical field of view of 45°.

In Figure 3.3 the red regions with an inference uncertainty of 10 $mm$ correspond to regions which have not been measured by the inspection, due to being out

\begin{figure}[h]
\centering
\subfigure[]{
\includegraphics[width=0.4\textwidth]{camera_laser.png}
\caption{Parameterization of the laser scanner setup geometry in RSL. The angle $\phi$ is defined w.r.t. the scan direction $\vec{d}$.}
\label{fig:camera_laser}
\end{figure}

\begin{figure}[h]
\centering
\subfigure[]{
\includegraphics[width=0.4\textwidth]{cylinder_head.png}
\caption{RSL interactive environment for the simulation of the constellation with parameters \{ $\theta = 84^\circ$, $\phi = 80^\circ$, $\tau = 5^\circ$, $dc = 0.4m$ \}.}
\label{fig:cylinder_head}
\end{figure}
of the field of view or not reached by the laser. Therefore, the prior standard deviation is not updated at those parts. Based on the lateral resolution and the uncertainty of the measurement at other regions, the surface uncertainty has been reduced.

The inference uncertainty along the surface can be considered as a suitable cost function for the optimization of the setup design, as this metric already takes the lateral resolution and the measurement uncertainty into account and indicates the degree of the informativeness of the inspection, in comparison to our prior knowledge about the surface. For the optimization, one can consider the sum of the uncertainties along the surface, or the maximum uncertainty, or other suitable statistics, and optimize the surface inspection parameters (both geometrical and optical) in an optimization framework.

4 Conclusion and Future Work

The probabilistic inference method discussed in this report fuses the measurement uncertainty and the resolution into a metric that indicates the adequacy of the inspection, by formulating the information a measurement delivers to us compared to our prior information about the surface. This means that the surface inference method actually derives the amount of new information that the measurement has delivered. This metric can be particularly useful to be used in an optimization framework to determine the setup degrees of freedom, by minimizing the a posteriori variance along the surface.

Our approach for the future work will be primarily to incorporate the proposed
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inference method in an optimization framework. In addition, we also attempt to
extend our model of the measurement uncertainty. The estimated measurement
uncertainties are based on modeling the sources of stochastic noise in the mea-
asurement; while other sources of error such as multiple reflections of the laser
on the surface are not considered. In the future, we will also attempt to approxi-
mate these sources of error in the simulations and take them into account for the
optimization.

Bibliography

[And14] Andrew Mchutchon. Nonlinear Modelling and Control using Gaus-

[Ble16] Blender Online Community. Blender - a 3D modelling and render-

[BLF15] Jürgen Beyerer, Fernando Puente León, and Christian Frese. Ma-
chine Vision: Automated Visual Inspection: Theory, Practice and

[BMI+16] Stephan Bergmann, Mahsa Mahammadikaji, Stephan Irgenfried,
Heinz Wörn, Jürgen Beyerer, and Carsten Dachsbacher. A Phe-
omenological Approach to Integrating Gaussian Beam Properties
and Speckle into a Physically-Based Renderer. In Matthias Hullin,
Marc Stamminger, and Tino Weinkauf, editors, Vision, Modeling &

Joint Workshop of Fraunhofer IOSB and Institute for Anthropomat-
ics, Vision and Fusion Laboratory, Karlsruher Schriften zur Anthropomatik, Triberg-Nussbach, Deutschland, July 2015. KIT Scientific
Publishing.

[BS87] Petr Beckmann and Andre Spizzichino. The scattering of electro-
magnetic waves from rough surfaces. Norwood, MA, Artech House,


<table>
<thead>
<tr>
<th>Reference</th>
<th>Authors</th>
<th>Title</th>
<th>Journal/Citation</th>
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