# An Optical Stokes Absolute Roll-angle Sensor with a Full Measurement Range of 360°


**Chia-Wei Chen[1, 2], Matthias Hartrumpf[2], Thomas Längle[2], Jürgen Beyerer[1, 2]**

1. Vision and Fusion Laboratory (IES), Karlsruhe Institute of Technology (KIT), Haid-und-Neu-Str. 7, 76131 Karlsruhe, Germany

2. Fraunhofer Institute of Optronics, System Technologies and Image Exploitation IOSB, Fraunhofer-straße 1, 76131 Karlsruhe, Germany


## Motivation

- Precise roll-angle measurement is often required in engineering and scientific instruments, e.g., object tracking,[1] motion of robotics and remote sensing.[2]
- In general, the polarization-based measurement can provide a large measuring range with a high resolution (subarcminute).[3]
- However, the unambiguous range of roll-angle measurements by conventional polarization-based methods is limited to 180°.[3]
- The proposed Stokes roll-angle sensor extended the unambiguous range to 360° by using a vortex retarder as a sensing unit.

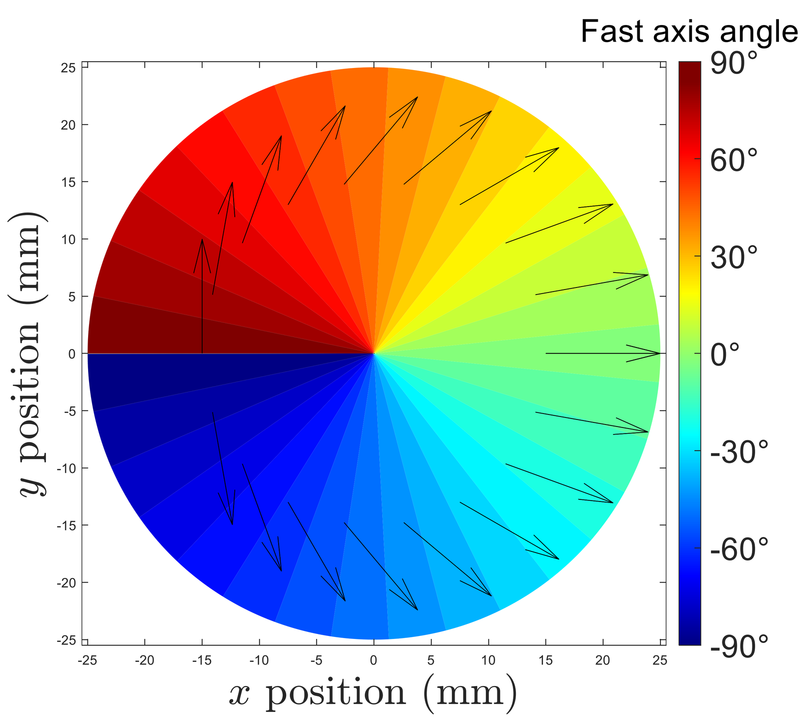
## Polarized light

The polarization effect of optical elements or interaction at boundaries can be described by Stokes vectors **S** and Mueller matrices **M**

- Stokes vector $\mathbf{S} \in \mathbb{R}^{4\times1}$
  - $\mathbf{S} = \begin{bmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{bmatrix} = \begin{bmatrix} I_x + I_y \\ I_x - I_y \\ I_{+45°} - I_{-45°} \\ I_R - I_L \end{bmatrix}$
- Normalized Stokes vector $\hat{\mathbf{S}}$
  - $\hat{\mathbf{S}} = \frac{\mathbf{S}}{S_0} = [1 \quad s_1 \quad s_2 \quad s_3]^{\mathrm{T}}$
- Mueller matrix $\mathbf{M} \in \mathbb{R}^{4\times4}$
  - $\mathbf{M} = \begin{bmatrix} M_{11} & M_{12} & M_{13} & M_{14} \\ M_{21} & M_{22} & M_{23} & M_{24} \\ M_{31} & M_{32} & M_{33} & M_{34} \\ M_{41} & M_{42} & M_{43} & M_{44} \end{bmatrix}$

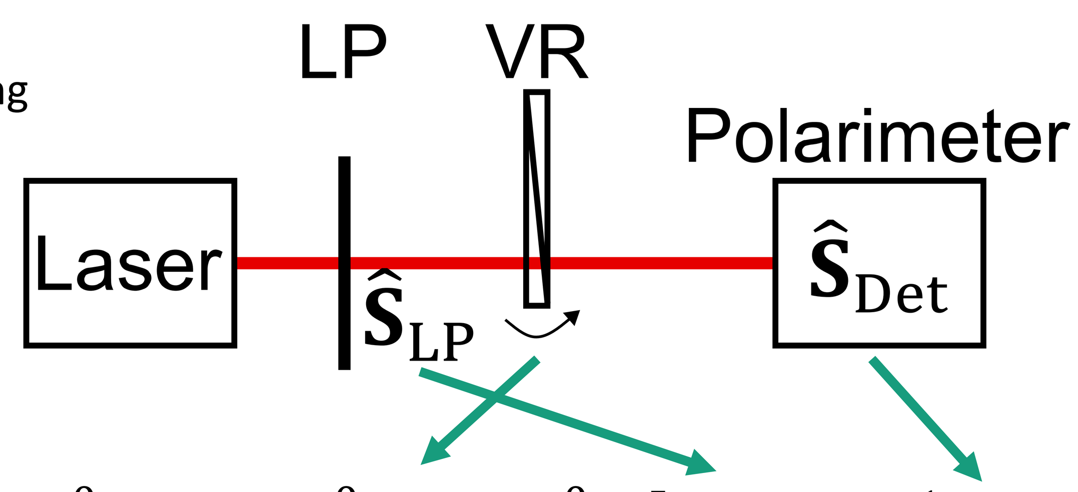
## Vortex retarder ($m = 1$)

- Fast axis angle: $\theta = \frac{m}{2}\varphi$
- Azimuthal angle $\varphi$ on the waveplate
- Retardance: $\frac{\pi}{4}$
- $\mathbf{M}_{\mathrm{RV}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos^2\varphi & \cos\varphi\sin\varphi & \sin\varphi \\ 0 & \cos\varphi\sin\varphi & \sin^2\varphi & -\cos\varphi \\ 0 & -\sin\varphi & \cos\varphi & 0 \end{bmatrix}$

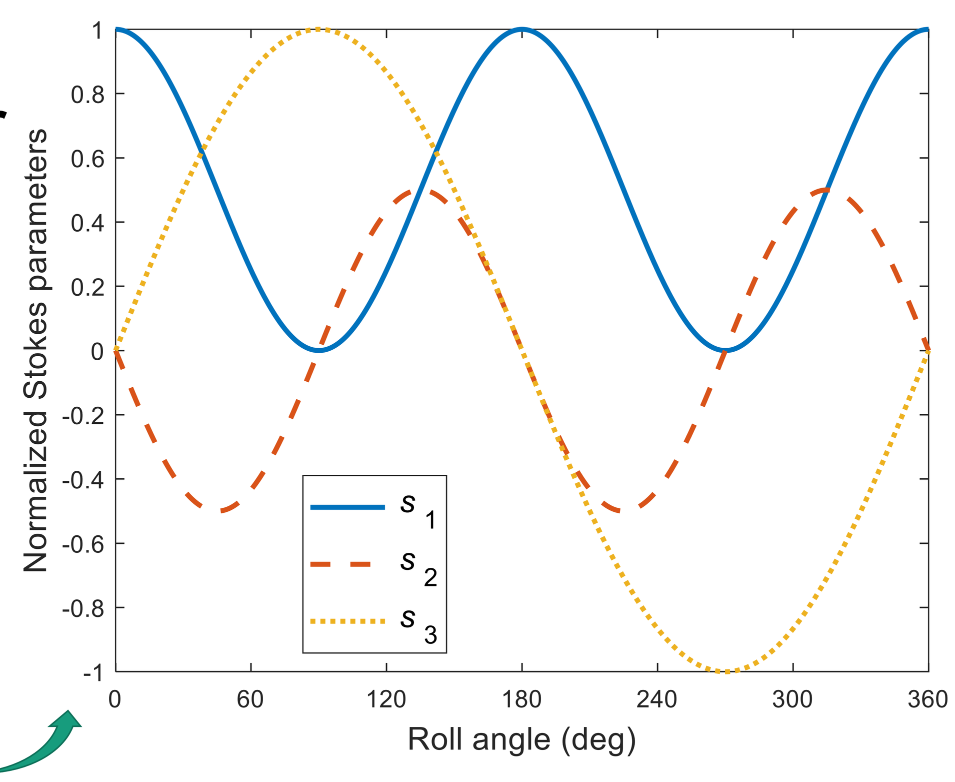



## Schematic of the Stokes Roll-angle Sensor

- $\hat{\mathbf{S}}_{\mathrm{Det}} = \mathbf{M}_{\mathrm{VR}} \cdot \hat{\mathbf{S}}_{\mathrm{LP}}$
  - Laser: 635 nm
  - Polarimeter: PAX1000VIS, Thorlabs Inc.
  - Rotary stage: K10CR1, Thorlabs Inc.
  - Vortex retarder (VR) with an offset of 6 mm
  - Linear polairzer (LP)

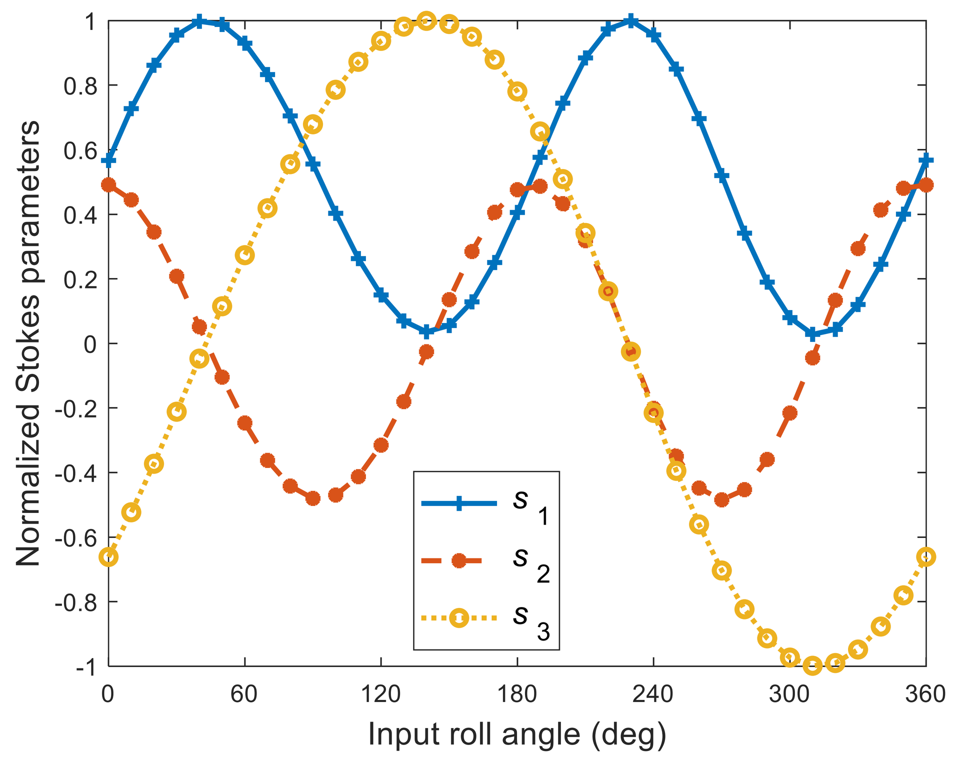


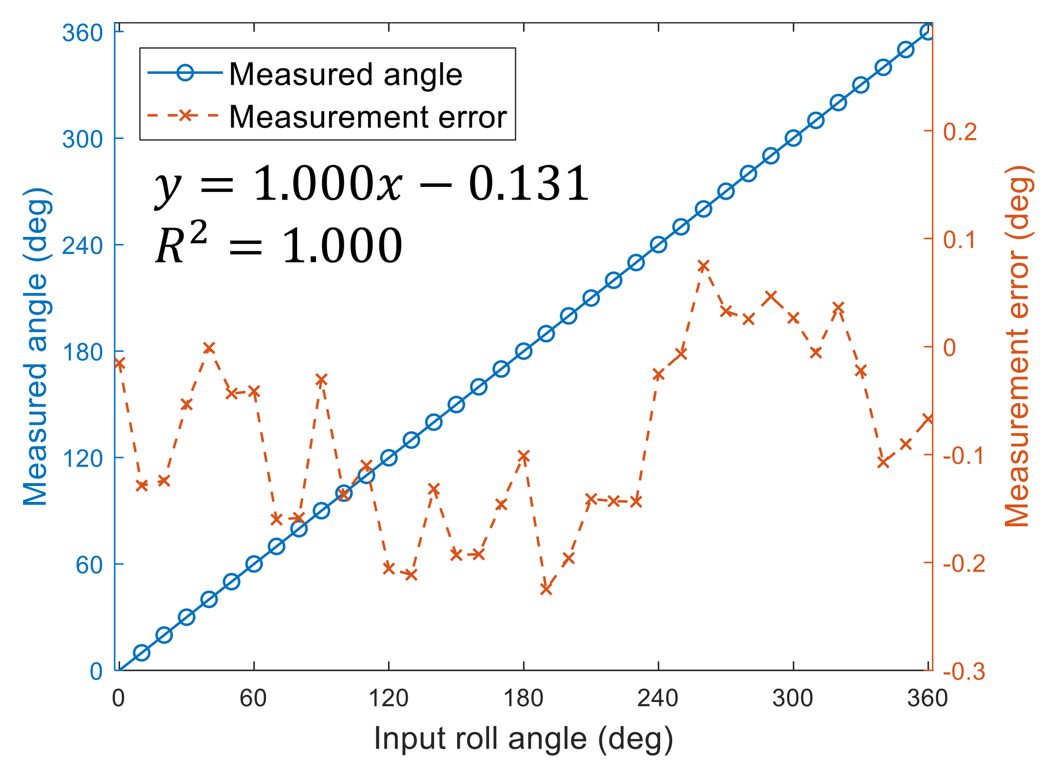

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos^2\varphi & \cos\varphi\sin\varphi & \sin\varphi \\ 0 & \cos\varphi\sin\varphi & \sin^2\varphi & -\cos\varphi \\ 0 & -\sin\varphi & \cos\varphi & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ \cos^2\varphi \\ \cos\varphi\sin\varphi \\ -\sin\varphi \end{bmatrix}$$




## Data Analysis and Expeimental Results

- Calibration
  - $s_i^{\mathrm{Cal}} = \sum_{j=1}^{5} a_j \sin(b_j\varphi + c_j)$
- Numerical fitting
  - $\chi^2(\varphi) = \sum_{i=1}^{3} (s_i^{\mathrm{Exp}} - s_i^{\mathrm{Cal}})^2$
  - $\mathrm{argmin}_{\varphi\in[0,360°)}\, \chi^2$
- Experimental results (5 measurements)
  - Root-mean-squared error: 0.12°




## Conclusion and Outlook

- A novel Stokes roll-angle sensor with a full measurement range of 360° is proposed.
- The unambiguous measuring range 360° is the largest in optical roll-sensing to date.
- The theoretical resolution is 0.01° (with a 14-bit encoder interpolation).
- Future work will be carried out to further improve the performance and stability of the proposed roll-angle sensor.


1 Jeong et al. "Polarized imaging interpreter for simultaneous clocking metrology of multiple objects." *Optics Letters* 46.19 (2021): 4992-4995.
2 Chen et al. "Remote Absolute Roll-Angle Measurement in Range of 180° Based on Polarization Modulation." *Nanomanufacturing and Metrology* 3 (2020): 228-235.
3 Chen et al. "Sensitivity enhanced roll-angle sensor by means of a quarter-waveplate." *tm-Technisches Messen* 88.s1 (2021): s48-s52.


Interaktive Echtzeitsysteme